M 250

4.2-3rd Review, Summary, Catch-up, Additional Practice

1) Intervals of increase, decrease.

2) 1^{st} derivative test for relative extrema
- critical values from f'
- sign chart for f'
- change of sign indicates extreme value
- max/min value is y-coord
- location is x-coord.

proof of an extreme value (relative)

3) Intervals of concavity

4) Inflection points $x = a$
- $f(a)$ must be defined

5) 2nd derivative test for relative extrema
- critical values from f' x_0
- no sign chart as a rule
- evaluate $f''(x_0)$

proof of an extreme value, if conclusive (relative)

6 ~~7~~) critical values when $f' = 0$ are called "stationary points."

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(1) $f(x) = (x-2)^3(x-1)$

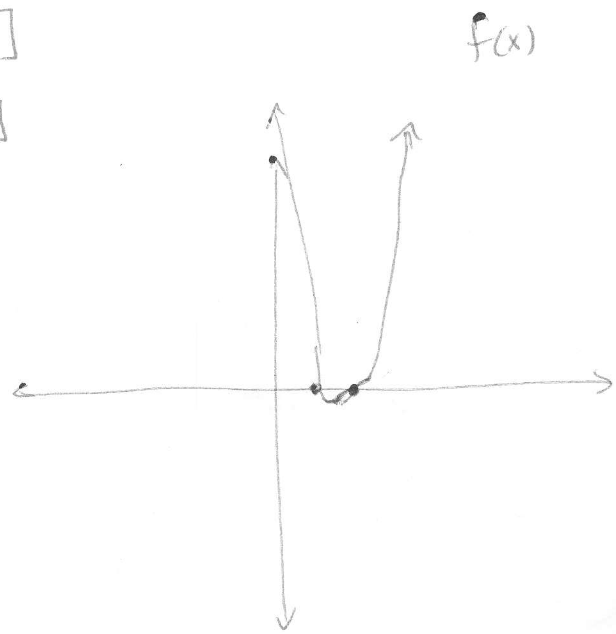
a) Find intervals of concavity & inflection points

b) Find C.V.s & use 2nd deriv test if possible

c) ~~Find~~ intervals of increase, decrease. Confirm rel. extrema using 1st deriv test

$$f'(x) = 3(x-2)^2(x-1) + (x-2)^3 \cdot 1$$
$$= 3(x-2)^2(x-1) + (x-2)^3$$
$$= (x-2)^2[3(x-1) + x-2]$$
$$= (x-2)^2[3x-3+x-2]$$
$$= (x-2)^2(4x-5)$$

$$f''(x) = 2(x-2)(4x-5) + (x-2)^2 \cdot 4$$
$$= 2(x-2)[4x-5+2(x-2)]$$
$$= 2(x-2)[4x-5+2x-4]$$
$$= 2(x-2)(6x-9)$$
$$= 6(x-2)(2x-3)$$



$f''(x) = 0 \quad x = 2 \quad x = \frac{3}{2}$

f'': (+) $\frac{3}{2}$ (−) 2 (+)

$\boxed{\text{concave up } (-\infty, \tfrac{3}{2}) \cup (2, \infty) \quad \text{Concave down } (\tfrac{3}{2}, 2)}$

$\boxed{\text{Inflection points: } (\tfrac{3}{2}, \tfrac{-1}{16}) \text{ and } (2, 0)}$

$f(\frac{3}{2}) = \frac{-1}{16}$

$f(2) = 0$

$f'(x) = 0$

$(x-2)^2(4x-5) = 0$

$\boxed{x = 2, \tfrac{5}{4} \text{ critical values}}$ both stationary pts. $f'(x)$ defined everywhere.

$f''(2) = 0$ $\boxed{\text{inconclusive 2nd derivative test for } x=2}$

$f''(\frac{5}{4}) = \frac{9}{4} > 0$ concave up $\boxed{\text{relative min at } x = \tfrac{5}{4}}$

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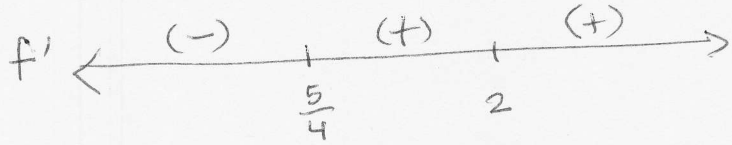
① con't

1st derivative test for C.V. $x=2$



$\boxed{\text{At } x=2 \text{ there is neither a relative max nor relative min}}$

$$f\left(\frac{5}{4}\right) = -\frac{27}{256}$$

$\boxed{\text{relative min } -\frac{27}{256} \text{ at } x=2}$

$\boxed{\begin{array}{ll} \text{Decreasing} & \left(-\infty, \frac{5}{4}\right] \\ \text{Increasing} & \left[\frac{5}{4}, \infty\right) \end{array}}$

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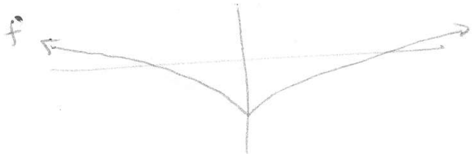
Find intervals where function is increasing and decreasing.

Use the first derivative test to find relative extrema.

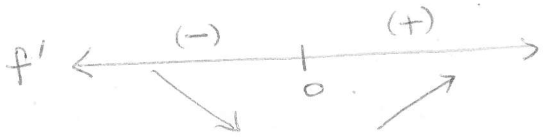
✓ (2) $f(x) = x^{2/3} - 4$



$f'(x) = x^{-1/3}$

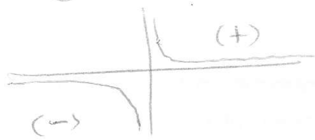
$f'(x) = 0$ nowhere

$f'(x)$ undef $x = 0$ ← C.V. but not a stationary point



$f'(-1) = \frac{1}{\sqrt[3]{-1}} = -1$ neg $\qquad f'(1) = \frac{1}{\sqrt[3]{1}} = 1$ pos

graph of $f'(x)$



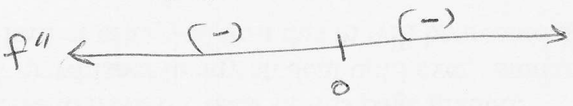
increasing $(0, \infty)$
decreasing $(-\infty, 0)$

Find intervals of concavity:

$f''(x) = -\frac{1}{3}x^{-4/3} = \frac{-1}{3\sqrt[3]{x^4}}$

$f''(x) \neq 0$

$f''(x)$ undef at $x = 0$



concave down everywhere $(-\infty, \infty)$

no inflection point

$f''(0)$ undefined → 2nd derivative test is inconclusive

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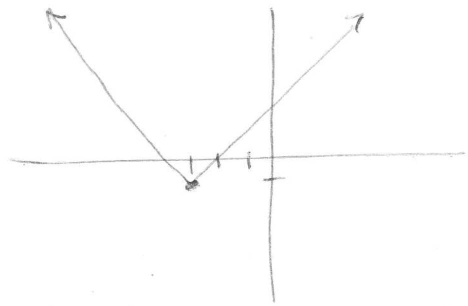
(2) cont.

Since f' changes sign from $(-)$ to $(+)$ at $x_0 = 0$, we conclude that $f(0) = 0^{2/3} - 4 = -4$ is a relative minimum at $x = 0$

$\boxed{\text{rel. min value } -4 \text{ occurs at } x = 0}$

✓ (3) $f(x) = |x+3| - 1$



$$= \begin{cases} x+3-1 & x+3 \geq 0 \\ -x-3-1 & x+3 < 0 \end{cases}$$

$$= \begin{cases} x+2 & x \geq -3 \\ -x-4 & x < -3 \end{cases}$$

$$f'(x) = \begin{cases} 1 & x > -3 \\ -1 & x < -3 \end{cases}$$

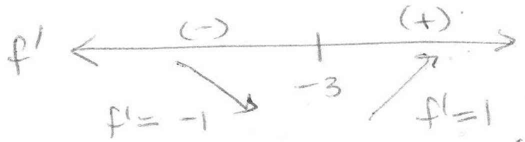
neither concave up nor down

$$f''(x) = \begin{cases} 0 & \forall x \neq -3 \\ \text{undef} & x = -3 \end{cases}$$

2nd deriv test inconclusive

$f'(x) \neq 0$ anywhere

$f'(x)$ does not exist at $x = -3$ ← critical value, but not a stationary point.

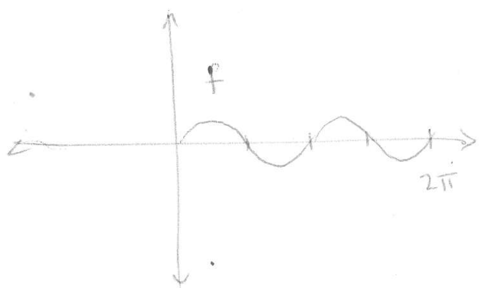


$f(-3) = -1$

$\boxed{\text{rel. min value } -1 \text{ at } x = -3}$

$\boxed{\begin{array}{l} \text{increasing } (-3, \infty) \\ \text{decreasing } (-\infty, -3) \end{array}}$

✓ (4) $f(x) = \sin x \cos x$ on $[0, 2\pi]$

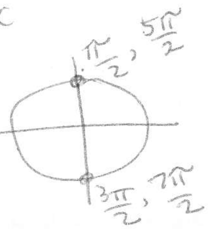


$f'(x) = \sin x(-\sin x) + \cos x(\cos x)$

$= -\sin^2 x + \cos^2 x$

$= \cos^2 x - \sin^2 x$

$= \cos(2x)$



$f'(x) = 0$ when

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ ← These critical values are all stationary points. cont →

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(5) con't

f': (+) on $(0, \frac{\pi}{4})$, (−) on $(\frac{\pi}{4}, \frac{3\pi}{4})$, (+) on $(\frac{3\pi}{4}, \frac{5\pi}{4})$, (−) on $(\frac{5\pi}{4}, \frac{7\pi}{4})$, (+) on $(\frac{7\pi}{4}, 2\pi)$

$0 \quad \frac{\pi}{4} \quad \frac{3\pi}{4} \quad \frac{5\pi}{4} \quad \frac{7\pi}{4} \quad 2\pi$

You can use the graph in your GC to save time in filling the sign chart. Graph the derivative. Where it's above the x-axis, it's positive; where it's below the x-axis, it's negative

f' changes sign at every critical values.

$$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$f\left(\frac{3\pi}{4}\right) = -\frac{1}{2}$$

$$f\left(\frac{5\pi}{4}\right) = \frac{1}{2}$$

$$f\left(\frac{7\pi}{4}\right) = -\frac{1}{2}$$

increasing f' (+)
$(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \frac{5\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

decreasing f' (−)
$(\frac{\pi}{4}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{7\pi}{4})$

relative min $-\frac{1}{2}$ at $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$

relative max $\frac{1}{2}$ at $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$

$$f''(x) = \frac{d}{dx}[\cos(2x)] = -\sin(2x)\cdot 2 = -2\sin(2x)$$

or $f''(x) = 2\cos x(-\sin x) - 2\sin x \cdot \cos x$
$= -2\cos x \sin x - 2\sin x \cos x$
$= -4\sin x \cos x$

$f''(x) = 0$ when $\sin x = 0$ or $\cos x = 0$
$x = 0, \pi, 2\pi$ $\quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2x = 0, \pi, 2\pi, 3\pi, 4\pi$
$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

f'': − on $(0, \frac{\pi}{2})$, + on $(\frac{\pi}{2}, \pi)$, − on $(\pi, \frac{3\pi}{2})$, + on $(\frac{3\pi}{2}, 2\pi)$

$0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$

concave down $(0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2})$

concave up $(\frac{\pi}{2}, \pi) \cup (\frac{3\pi}{2}, 2\pi)$

inflection points.
$(0,0)$
$(\frac{\pi}{2}, 0)$
$(\pi, 0)$
$(\frac{3\pi}{2}, 0)$
$(2\pi, 0)$

$f''(\frac{\pi}{4}) < 0$ (−) rel max
$f''(\frac{3\pi}{4}) > 0$ (+) rel min
$f''(\frac{5\pi}{4}) < 0$ (−) rel max
$f''(\frac{7\pi}{4}) > 0$ (+) rel min

conclusive, could be used instead of 1st derivative test

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(5) $f(x) = \dfrac{x^2}{x^2-9}$

a) Find the critical values.
b) Find the intervals where the function is increasing and decreasing
c) Find local extrema using the first derivative test.
d) Find the intervals where f is concave up and concave down.
e) Find the points of inflection.
f) Find the local extrema using the second derivative test.

a) $f'(x) = \dfrac{(x^2-9)\cdot 2x - x^2(2x)}{(x^2-9)^2}$ quotient rule

$= \dfrac{2x^3 - 18x - 2x^3}{[(x-3)(x+3)]^2}$ dist
factor diff of sq.

$= \dfrac{-18x}{(x-3)^2(x+3)^2}$ combine like terms
simplify factors

$f'(x) = 0$

$\dfrac{-18x}{(x-3)^2(x+3)^2} = 0$

mult by $(x-3)^2(x+3)^2$

$-18x = 0$

$\boxed{x = 0 \quad \text{c.v.}}$

$f'(x)$ undef

$(x-3)^2(x+3)^2 = 0$

$x = 3, -3$ not c.v.s because $f(3)$ and $f(-3)$ undefined.

b) f' $\longleftarrow$ (+) | (+) | (−) | (−) $\longrightarrow$

at -3, 0, 3

$\boxed{\begin{array}{l}\text{increasing } (-\infty,-3) \cup (-3,0) \\ \text{decreasing } (0,3) \cup (3,\infty)\end{array}}$

c) Using sign chart in b), change of sign at $x=0$. + ↗ − ↘

$f(0) = \dfrac{0^2}{0^2-9} = 0$

$\boxed{\text{local maximum } 0 \text{ at } x=0}$

d) $f''(x) = \dfrac{(x^2-9)^2(-18) - (-18x)\cdot 2(x^2-9)(2x)}{(x^2-9)^4}$ quotient rule

$= \dfrac{-18(x^2-9)\left[(x^2-9) - 4x^2\right]}{(x^2-9)^4}$ factor $-18(x^2-9)$

$= \dfrac{-18(-3x^2-9)}{(x^2-9)^3}$ cancel (x^2-9)
combine

$f''(x) = \dfrac{54(x^2+3)}{(x^2-9)^3}$

$f''(x) = \dfrac{54(x^2+3)}{(x-3)^3(x+3)^3}$

f'': (+) on $(-\infty, -3)$, (−) on $(-3, 3)$, (+) on $(3, \infty)$

-3 3

$f''(x) = 0$
$54(x^2+3) = 0$
no real solutions

concave up $(-\infty, -3) \cup (3, \infty)$
Concave down $(-3, 3)$

e) f'' changes sign at $x = -3$ and $x = 3$
<u>but</u> $f(3)$ and $f(-3)$ are undefined.

no inflection points

f) Test C.V. in f'':

$f''(0) = \dfrac{54(0^2+3)}{(0^2-9)^3} = \dfrac{54(3)}{(-9)^3} < 0$ concave down

local maximum at $x = 0$

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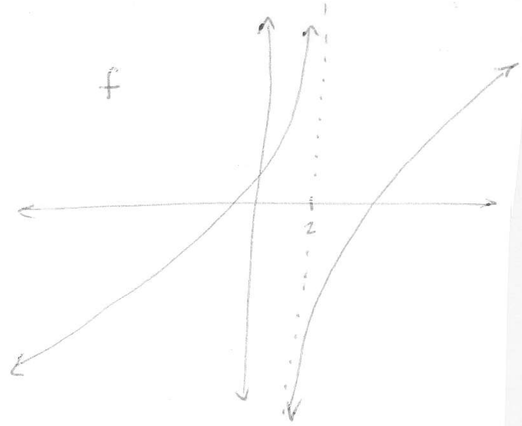
✓ (6) $f(x) = \dfrac{x^2-3x-4}{x-2} = (x^2-3x-4)(x-2)^{-1}$



Method 1
Product Rule

$f'(x) = (x^2-3x-4)(-1)(x-2)^{-2} + (2x-3)(x-2)^{-1}$

$= (x-2)^{-2}\left[-x^2+3x+4 + (2x-3)(x-2)\right]$

$= (x-2)^{-2}\left[-x^2+3x+4 + 2x^2-7x+6\right]$

$= (x-2)^{-2}\left[x^2-4x+10\right] = \dfrac{x^2-4x+10}{(x-2)^2}$

$f'(x)=0 \qquad x^2-4x+10=0$

$x = \dfrac{4 \pm \sqrt{16-4(10)}}{2}$ = imaginary, no stationary points

$f'(x)$ undef $x=2$ critical value, but not a stationary value

f': $(+)$ on $(-\infty, 2)$ ↗, $(+)$ on $(2, \infty)$ ↗

since $f'(x) > 0$ at all values, f increases everywhere
$x=2$ is neither a relative max nor relative min.

$\boxed{\text{f has no relative extrema}}$

$f''(x) = (x-2)^{-2}(2x-4) + (x^2-4x+10)(-2)(x-2)^{-3}$

$= 2(x-2)^{-3}\left[(x-2)(x-2) - (x^2-4x+10)\right]$

$= \dfrac{2(x^2-4x+4-x^2+4x-10)}{(x-2)^3}$

$= \dfrac{-12}{(x-2)^3}$

$f''(x) \neq 0$
$f''(x)$ undef at $x=2$

f'': $+$ on $(-\infty, 2)$, $-$ on $(2, \infty)$

$f(2)$ not defined, so $x=2$ is <u>not</u> an inflection point.

$\boxed{\begin{array}{ll} \text{concave up} & (-\infty, 2) \\ \text{concave down} & (2, \infty) \end{array}}$

$f''(2)$ undef $\rightarrow$ 2nd derivative test is inconclusive.

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cont

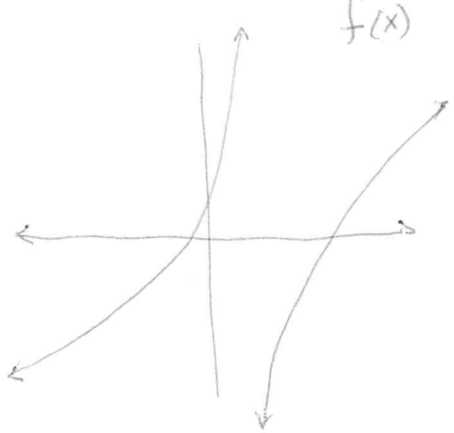
(5) $f(x) = \dfrac{x^2-3x-4}{x-2}$

$f'(x) = \dfrac{(x-2)(2x-3) - (x^2-3x-4)\cdot 1}{(x-2)^2}$

Method 2:
Quotient Rule

$= \dfrac{2x^2-7x+6 - x^2+3x+4}{(x-2)^2}$

$= \dfrac{x^2-4x+10}{(x-2)^2}$

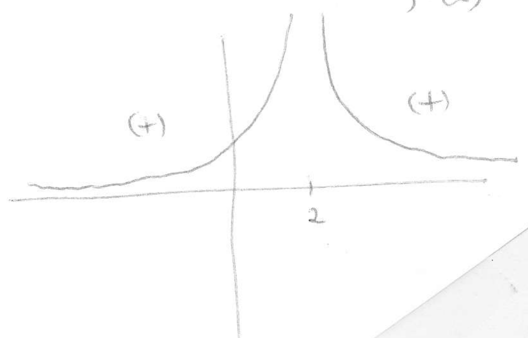


$f'(x) = 0$ when $x^2-4x+10=0$

$x = \dfrac{4 \pm \sqrt{16-4(10)}}{2}$

$x = \dfrac{4 \pm \sqrt{-24}}{2}$ not real
not on graph

$f'(x)$ undef $\boxed{x=2 \text{ critical value}}$



f' $\xleftarrow{\quad + \quad} \big| \xrightarrow{\quad + \quad}$
2

f is increasing everywhere

$\boxed{\text{increasing } (-\infty, \infty)}$

$f''(x) = \dfrac{(x-2)^2(2x-4) - (x^2-4x+10)\cdot 2(x-2)}{(x-2)^4}$

$= \dfrac{2(x-2)\left[(x-2)(x-2) - (x^2-4x+10)\right]}{(x-2)^4}$

$= \dfrac{2\left[x^2-4x+4-x^2+4x-10\right]}{(x-2)^3}$

$= \dfrac{2(-6)}{(x-2)^3} = \dfrac{-12}{(x-2)^3}$

Concavity and inflection as with product rule method.

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a) Determine the intervals on which the function is concave up or concave down

b) Find all points of inflection.

c) Find all critical values.

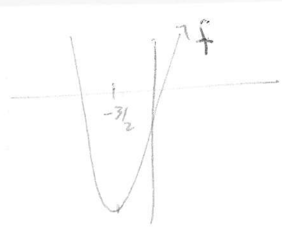
d) Use the second derivative test, if possible to determine relative extrema. When the second derivative test is inconclusive, so state, and use the first derivative test.

7) $f(x) = x^2 + 3x - 8$

$f'(x) = 2x + 3$

$f''(x) = 2 > 0$ always

$\boxed{\text{concave up } (-\infty, \infty) \quad \text{no inflection points}}$



$f'(x) = 0 \qquad 2x + 3 = 0$

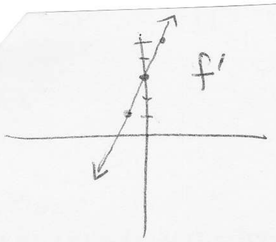
$\boxed{x = -\frac{3}{2} \quad \text{critical value}}$ is a stationary value

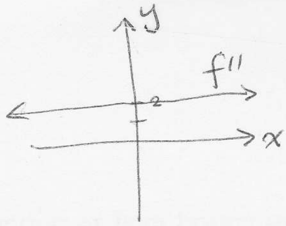
$f''(x) = 2 > 0$ concave up everywhere

relative min

$f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 8 = -\frac{41}{4}$

$\boxed{\text{rel min } -\frac{41}{4} \text{ at } x = -\frac{3}{2}}$





Note: Critical values, where relative extrema may occur, are always found using the first derivative.

f' (−) $-3/2$ (+)

$\boxed{\begin{array}{l}\text{increasing } (-\infty, -3/2] \\ \text{decreasing } [-3/2, \infty)\end{array}}$

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8

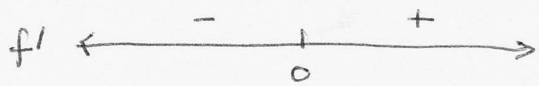
$$f(x) = \frac{x^2}{x^2+1}$$

Method 1 Quotient Rule

$$f'(x) = \frac{(x^2+1)2x - x^2(2x)}{(x^2+1)^2}$$

$$= \frac{2x\left[x^2+1-x^2\right]}{(x^2+1)^2}$$

$$= \frac{2x}{(x^2+1)^2}$$



$$\boxed{\begin{array}{l}\text{decreasing } (-\infty, 0] \\ \text{increasing } [0, \infty)\end{array}}$$

1st derivative test also shows $x=0$ is location of relative min.

$$f''(x) = \frac{(x^2+1)^2 \cdot 2 - 2x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$$

$$f''(x) = \frac{2(x^2+1)\left[x^2+1-4x^2\right]}{(x^2+1)^4}$$

$$= \frac{2(-3x^2+1)}{(x^2+1)^3}$$

$f''(x) = 0$ when $-3x^2+1=0$

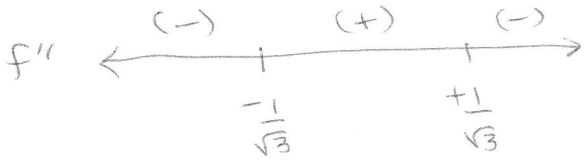
$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\frac{1}{\sqrt{3}}$$

$f''(x)$ undef when $x^2+1=0$

imaginary



$$\boxed{\begin{array}{l}\text{concave up } \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \\ \text{concave down } \left(-\infty, \frac{-1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right)\end{array}}$$

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(8) cont

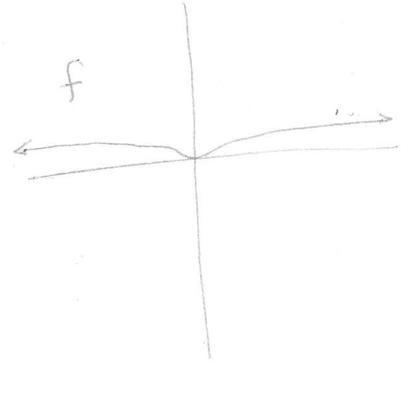
$f\left(\frac{-1}{\sqrt{3}}\right) = \frac{1}{4}$

$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{4}$

$\boxed{\text{inflection points } \left(\frac{-1}{\sqrt{3}}, \frac{1}{4}\right) \text{ and } \left(\frac{1}{\sqrt{3}}, \frac{1}{4}\right)}$

Method 2: Quotient Rule

$f(x) = \frac{x^2}{x^2+1} = x^2(x^2+1)^{-1}$



$f'(x) = x^2(-1)(x^2+1)^{-2}(2x) + 2x(x^2+1)^{-1}$

$= -2x^3(x^2+1)^{-2} + 2x(x^2+1)^{-1}$

$= -2x(x^2+1)^{-2}[x^2 - (x^2+1)]$

$= -2x(x^2+1)^{-2}(-1)$

$= 2x(x^2+1)^{-2}$

$f''(x) = 2x(-2)(x^2+1)^{-3}(2x) + 2(x^2+1)^{-2}$

$= -8x^2(x^2+1)^{-3} + 2(x^2+1)^{-2}$

$= -2(x^2+1)^{-3}[4x^2 - (x^2+1)]$

$= -2(x^2+1)^{-3}(3x^2-1)$

critical values $f'(x) = 0$ $2x = 0$ $\boxed{x = 0 \text{ stationary c.v.}}$

$f'(x)$ undef imaginary

$f''(0) = \frac{-2(3 \cdot 0 - 1)}{(0^2+1)^3} = 2 > 0$ MIN

2nd derivative test is conclusive.

$f(0) = 0$

$\boxed{\text{rel min } 0 \text{ at } x = 0}$

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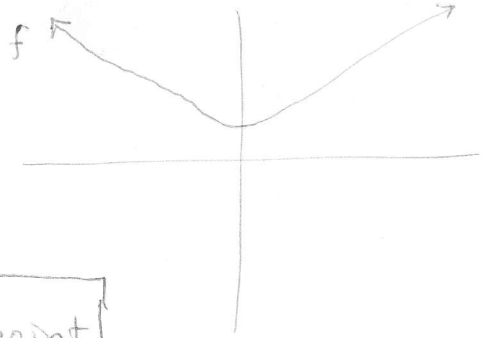


$f(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$

$f'(x) = \frac{1}{2}(x^2+1)^{-1/2}(2x)$

$= \dfrac{x}{\sqrt{x^2+1}} = x(x^2+1)^{-1/2}$

$f'(x)=0$ when $\boxed{x=0}$ critical value stationary point

$f'(x)$ undefined imaginary

$f''(x) = x\cdot\left(-\frac{1}{2}\right)(x^2+1)^{-3/2}\cdot 2x + 1\cdot(x^2+1)^{-1/2}$

$= -x^2(x^2+1)^{-3/2} + (x^2+1)^{-1/2}$

$= (x^2+1)^{-3/2}\left[-x^2+x^2+1\right]$

$= (x^2+1)^{-3/2}$

$= \dfrac{1}{\sqrt{(x^2+1)^3}}$

$\boxed{\text{concave up } (-\infty,\infty)}$

$\boxed{\text{no inflection point}}$

$f''(0) = \dfrac{1}{\sqrt{(0+1)^3}} = 1 > 0$ pos. + concave up at $x=0$.

$f(0) = \sqrt{0^2+1} = 1$

$\boxed{\text{rel min } 1 \text{ at } x=0}$

f': $(-)$ on the left of 0, $(+)$ on the right of 0

$\boxed{\begin{array}{l}\text{decreasing } (-\infty, 0] \\ \text{increasing } [0, \infty)\end{array}}$

1st derivative test confirms relative min at $x=0$.

Math 250

(10) $f(x) = \dfrac{x}{x-1}$

Method 1) Quotient Rule

$f'(x) = \dfrac{(x-1)\cdot 1 - x(1)}{(x-1)^2}$

$= \dfrac{x-1-x}{(x-1)^2}$

$f'(x) = \dfrac{-1}{(x-1)^2}$

$f'(x) = -(x-1)^{-2}$

$f''(x) = 2(x-1)^{-3}$

$f''(x) = 0$ nowhere

$f''(x)$ undef $x=1$

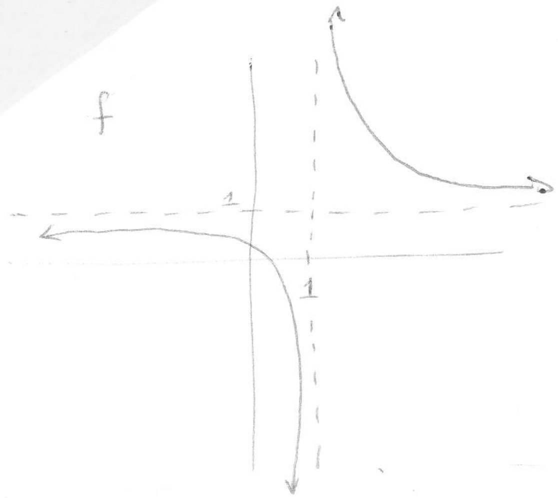
f'' ← (−) | 1 (+) →

$\boxed{\text{concave up } (1,\infty) \quad \text{concave down } (-\infty,1)}$

$f(1)$ undef

not an inflection pt.

$\boxed{\text{no inflection points}}$



$f(x) = \dfrac{x}{x-1} = x(x-1)^{-1}$

Method 2: Product Rule

$f'(x) = x(-1)(x-1)^{-2} + 1\cdot(x-1)^{-1}$

$= (x-1)^{-2}\left[-x + x - 1\right]$

$= -(x-1)^{-2}$

$f''(x) = 2(x-1)^{-3}$

$f'(x) = 0$ no

$f'(x)$ undef $x=1$

$\boxed{x=1 \text{ critical value}}$ not stationary

$f''(1) = 2(1-1)^{-3} = 0$ inconclusive

Back to 1st derivative test

f' ← (−) | 1 (−) →

$x=1$ is neither a max nor a min

$\boxed{f \text{ has no relative extrema}}$

$\boxed{\text{decreasing } (-\infty,1)\cup(1,\infty)}$

f not defined at $x=1$

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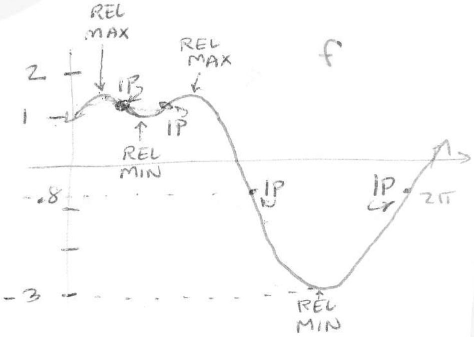
Approximate to nearest ten-thousandth.



10) $f(x) = 2\sin x + \cos(2x)$ on $[0, 2\pi]$

$f'(x) = 2\cos x - \sin(2x) \cdot 2$

$= 2\cos x - 2\sin(2x)$

$f''(x) = -2\sin x - 2\cos(2x) \cdot 2$

$= -2\sin x - 4\cos(2x) = -2(\sin x + 2\cos(2x))$

DO NOT DO CONCAVITY + INFLECTION PTS IN CLASS!!!

$f''(x) = 0$

$-2\sin x - 4\cos(2x) = 0$

$-2\sin x - 4(1 - 2\sin^2 x) = 0$ substitute identity

$\frac{-2\sin x}{\div 2} - \frac{4}{2} + \frac{8\sin^2 x}{2} = \frac{0}{2}$

$-\sin x - 2 + 4\sin^2 x = 0$

$4\sin^2 x - \sin x - 2 = 0$ subst $u = \sin x$ if desired.

$4u^2 - u - 2 = 0$

-8, -1: $4u^2 - u - 2$ does not factor!

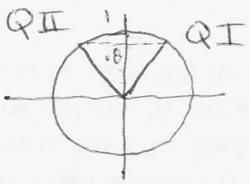
Off to the quadratic formula!

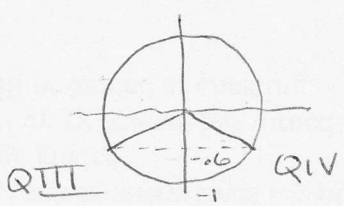
$$u = \frac{1 \pm \sqrt{1 - 4(4)(-2)}}{2(4)}$$

$$u = \frac{1 \pm \sqrt{1 + 32}}{8}$$

$\sin x = \frac{1 + \sqrt{33}}{8}$ $\qquad$ $\sin x = \frac{1 - \sqrt{33}}{8}$ $\qquad$ These angles exist, but they are not multiples of $\frac{\pi}{6}$ or $\frac{\pi}{4}$.

$\sin x \approx .8431$ $\qquad$ $\sin x = -.5931$





Nasty, real-world angles...

$x = \sin^{-1}\left(\frac{1 + \sqrt{33}}{8}\right)$ $\qquad$ $x = \sin^{-1}\left(\frac{1 - \sqrt{33}}{8}\right)$

$x \approx 1.0030$ radians $\qquad$ $x \approx -.6349$ ← not in $[0, 2\pi]$, positive

$x = \pi - \sin^{-1}\left(\frac{1 + \sqrt{33}}{8}\right)$ $\qquad$ $x = \sin^{-1}\left(\frac{1 - \sqrt{33}}{8}\right) + 2\pi$ ← coterminal angle

$x \approx 2.1386$ radians $\qquad$ $x \approx 5.6483$ radians

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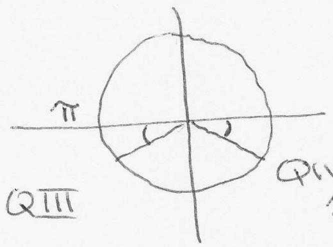
To find the QIII angle, notice two congruent angles

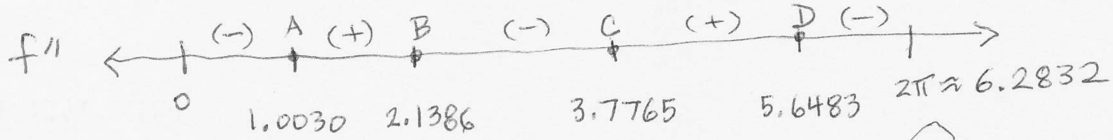
$x \approx .6349$ radians if position is not considered.



QIII angle $x = \pi - \sin^{-1}\left(\frac{1-\sqrt{33}}{8}\right)$

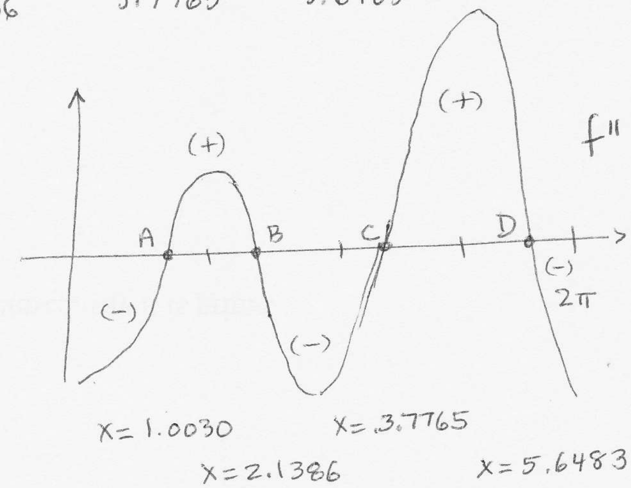
$x \approx \pi + .6349$

$\boxed{x \approx 3.7765 \text{ radians}}$



Graph of f''

GC zero to check values ☺



$\boxed{\begin{array}{l}\text{concave up } f''(x)>0 \\ (1.0030, 2.1386) \cup (3.7765, 5.6483)\end{array}}$

$\boxed{\begin{array}{l}\text{concave down } f''(x)<0 \\ (0, 1.0030) \cup (2.1386, 3.7765) \cup (5.6483, 2\pi)\end{array}}$

All four of the zeros of f'' are inflection points !!!!

To get y-coords, evaluate in original function.

To make this easier, store each one in GC memory

A: $\sin^{-1}\left(\frac{1+\sqrt{33}}{8}\right)$ $\boxed{\text{STO>}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATH}}$ A

B: $\pi - \sin^{-1}\left(\frac{1+\sqrt{33}}{8}\right)$ $\boxed{\text{STO>}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{APPS}}$ B

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C: $\pi - \sin^{-1}\left(\frac{1-\sqrt{33}}{8}\right)$ $\boxed{\text{STO>}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{PRGM}}$ C

D: $\sin^{-1}\left(\frac{1-\sqrt{33}}{8}\right) + 2\pi$ $\boxed{\text{STO>}}$ $\boxed{\text{ALPHA}}$ $\boxed{x^{-1}}$ D

$y_1 = 2\sin(x) + \cos(2x)$

$\boxed{\text{VARS}}$ $\rightarrow$ $\boxed{\text{Y-VARS}}$

1. Function
   1. Y_1

$Y_1($ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATH}}$ A $)$ $\boxed{\text{ENTER}}$

use $\boxed{(}$ key   use $\boxed{)}$ key

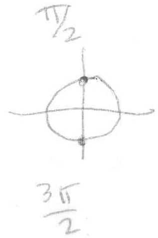
Approximate INFLECTION POINTS
$(1.0030,\ 1.2646)$
$(2.1386,\ 1.2646)$
$(3.7765,\ -.8896)$
$(5.6483,\ -.8896)$

$f'(x) = 0$ $2\cos x - 2\sin(2x) = 0$

$2\cos x - 2[2\sin x\cos x] = 0$ trig identity

$2\cos x - 4\sin x\cos x = 0$

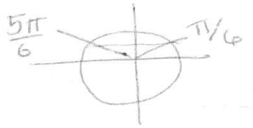
$2\cos x(1 - 2\sin x) = 0$ factor



$\cos x = 0$ $1 - 2\sin x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$



$$\boxed{\text{critical values} \quad x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}}$$

all are stationary points

f': on $(0, \frac{\pi}{6})$ $+$ $\nearrow$; on $(\frac{\pi}{6}, \frac{\pi}{2})$ $-$ $\searrow$; on $(\frac{\pi}{2}, \frac{5\pi}{6})$ $+$ $\nearrow$; on $(\frac{5\pi}{6}, \frac{3\pi}{2})$ $-$ $\searrow$; on $(\frac{3\pi}{2}, 2\pi)$ $+$ $\nearrow$

1st derivative test shows rel mins at $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$

rel maxes at $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$.

M250

cont

$f''\left(\frac{\pi}{2}\right) = -2\left(\sin \frac{\pi}{2} + 2\cos \pi\right)$

$= -2\left(1 + 2(-1)\right)$

$= 2 > 0$ MIN

$f''\left(\frac{3\pi}{2}\right) = -2\left(\sin \frac{3\pi}{2} + 2\cos 3\pi\right)$

$= -2\left(-1 + 2(-1)\right)$

$= 6 > 0$ MIN

$f''\left(\frac{\pi}{6}\right) = -2\left(\sin \frac{\pi}{6} + 2\cos \frac{\pi}{3}\right)$

$= -2\left(\frac{1}{2} + 2\left(\frac{1}{2}\right)\right)$

$= -3 < 0$ MAX

$f''\left(\frac{5\pi}{6}\right) = -2\left(\sin \frac{5\pi}{6} + 2\cos \frac{5\pi}{3}\right)$

$= -2\left(\frac{1}{2} + 2\left(\frac{1}{2}\right)\right)$

$= -3 < 0$ MAX

2nd derivative test is conclusive.

$f\left(\frac{\pi}{2}\right) = 2\sin\left(\frac{\pi}{2}\right) + \cos \pi$

$= 2 \cdot 1 + (-1) = 1$ $\boxed{\text{rel min } 1 \text{ at } x = \frac{\pi}{2}}$

$f\left(\frac{3\pi}{2}\right) = 2\sin\left(\frac{3\pi}{2}\right) + \cos 3\pi$

$= 2 \cdot (-1) + (-1) = -3$ $\boxed{\text{rel min } -3 \text{ at } x = \frac{3\pi}{2}}$

$f\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{3}\right)$

$= 2 \cdot \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$ $\boxed{\text{rel max } \frac{3}{2} \text{ at } x = \frac{\pi}{6}}$

$f\left(\frac{5\pi}{6}\right) = 2\sin \frac{5\pi}{6} + \cos \frac{5\pi}{3}$

$= 2\left(\frac{1}{2}\right) + \frac{1}{2} = \frac{3}{2}$ $\boxed{\text{rel max } \frac{3}{2} \text{ at } x = \frac{5\pi}{6}}$

check graph on GC to confirm.